Trigonometric functions-graphics (part II)

In the previous file (**trigonometric functions-graphics (part II)** we studied how to draw graphics, depending on numbers *a, b* and *c*. Now we can make the whole graph $y = a\sin(bx + c)$.

PROCEDURE::

i) **Draw the graph of** $y = \sin x$

ii) **Spot numbers *a, b* and *c*, and find** $T = \frac{2\pi}{b}$**. construct** $y = \sin bx$**.**

iii) **Determine the value of expression** $\frac{c}{b}$ **and construct** $y = \sin(bx + c)$

iv) **Amplitude value *a* helps us to draw the final graph** $y = a\sin(bx + c)$

This is one way of drawing graphics. The second is the direct examination of important points, and we have already mentioned that here you need to know how to solve trigonometric equations.

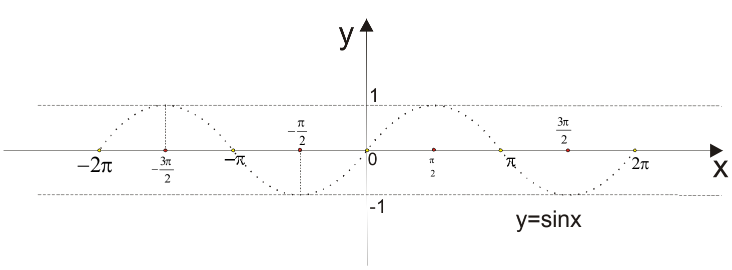
Example 1. **Draw a graph** $y = 3\sin(2x + \frac{\pi}{4})$

<u>**Solution:**</u>

<u>***first way***</u>

From $y = 3\sin(2x + \frac{\pi}{4})$ are $a = 3, b = 2, c = \frac{\pi}{4}$

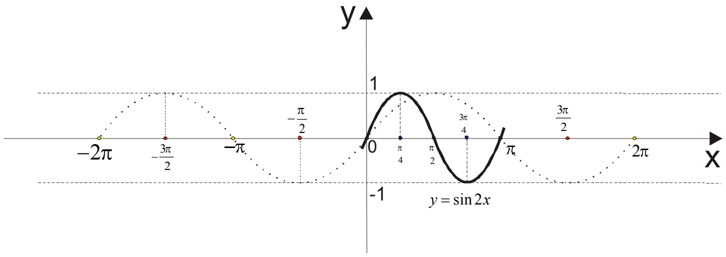
First we draw graph of basic function $y = \sin x$.



slika 1.

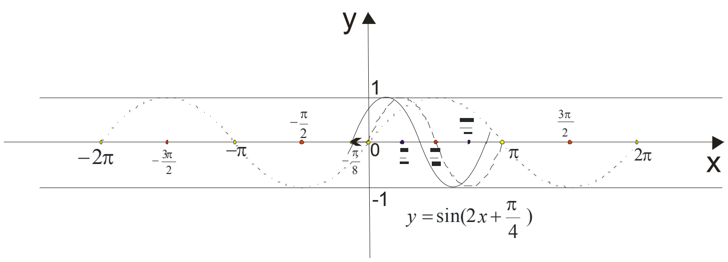
Find a period: $T = \frac{2\pi}{b} \rightarrow T = \frac{2\pi}{2} \rightarrow \boxed{T = \pi}$

Further draw graph $y = \sin 2x$



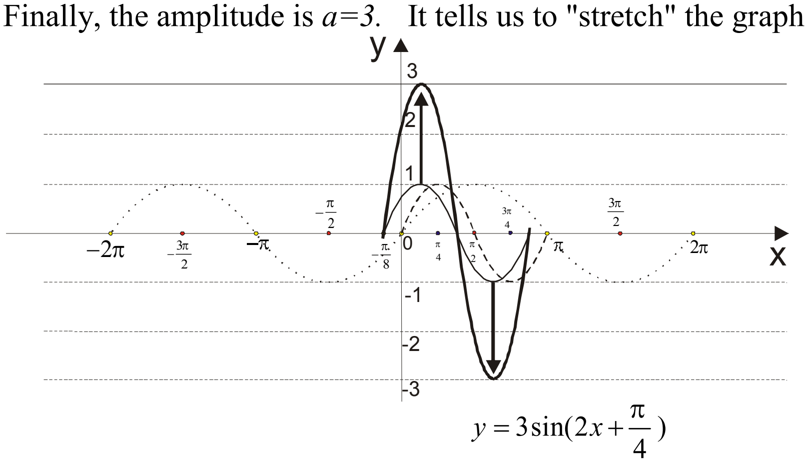
slika 2.

Value of expression $\frac{c}{b}$ is $\frac{c}{b} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8}$**.** We're moving graphic $y = \sin 2x$ for $\frac{\pi}{8}$ **left**:



slika 3.

Finally, the amplitude is *a=3*. It tells us to "stretch" the graph between -3 and 3 along the y-axis



slika 4.

second way

Write down the value of *a, b* and *c, and* find $T = \frac{2\pi}{b}$**.**

Investigate where are the zero function.

We are looking points of maximum and minimum

$a = 3, b = 2, c = \frac{\pi}{4}$ and $T = \frac{2\pi}{b} \to T = \frac{2\pi}{2} \to \boxed{T = \pi}$

Zero function

These are places where the graph cuts x axis.

$y = 0$

$3\sin(2x + \frac{\pi}{4}) = 0$

$\sin(2x + \frac{\pi}{4}) = 0 \to 2x + \frac{\pi}{4} = 0 \vee 2x + \frac{\pi}{4} = \pi$

$2x + \frac{\pi}{4} = 0$

$2x = -\frac{\pi}{4} \to \boxed{x = -\frac{\pi}{8}}$ Here you can add a period (T=π): $\boxed{x = -\frac{\pi}{8} + k\pi}$ $k \in Z$

$2x + \frac{\pi}{4} = \pi$

$2x = \frac{3\pi}{4} \to \boxed{x = \frac{3\pi}{8}} \to \boxed{x = \frac{3\pi}{8} + k\pi}$ $k \in Z$

These points are at the x axis.

Maximum

As the amplitude is *a=3*.The function will have a maximum value of $y = 3$.

$y = 3$

$3\sin(2x + \frac{\pi}{4}) = 3$

$\sin(2x + \frac{\pi}{4}) = 1 \to 2x + \frac{\pi}{4} = \frac{\pi}{2} \to 2x = \frac{\pi}{2} - \frac{\pi}{4} \to 2x = \frac{\pi}{4} \to \boxed{x = \frac{\pi}{8}}$

And here we have to add the period: $\boxed{x = \frac{\pi}{8} + k\pi}$ $k \in Z$

Minimum

We have a minimum value for $y = -3$

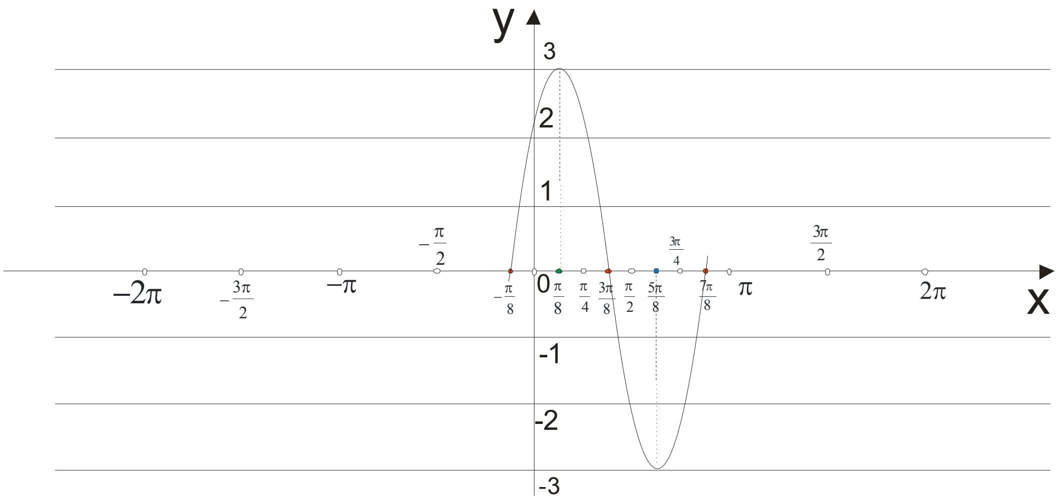
$y = -3$

$3\sin(2x + \frac{\pi}{4}) = -3$

$\sin(2x + \frac{\pi}{4}) = -1 \to 2x + \frac{\pi}{4} = \frac{3\pi}{2} \to 2x = \frac{3\pi}{2} - \frac{\pi}{4} \to 2x = \frac{5\pi}{4} \to \boxed{x = \frac{5\pi}{8}}$

add the period: $\boxed{x = \frac{5\pi}{8} + k\pi}$ $k \in Z$

Now put together a graph:

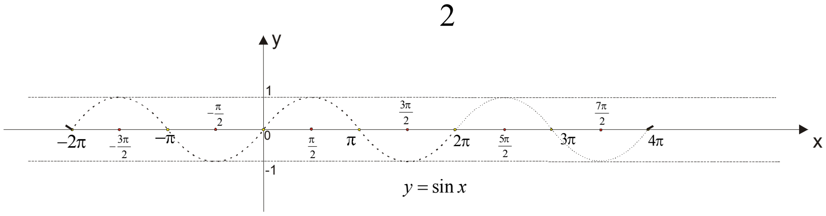


$$y = 3\sin(2x + \frac{\pi}{4})$$

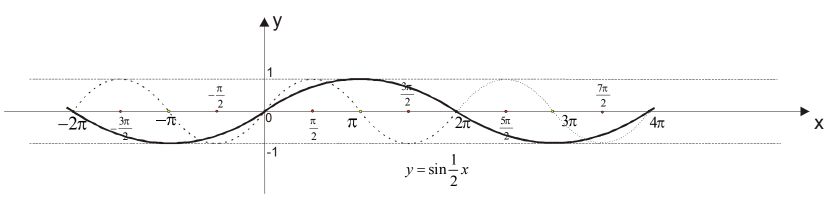
Example 2. Draw a graph $y = -2\sin(\frac{1}{2}x + \frac{\pi}{6})$

Solution:

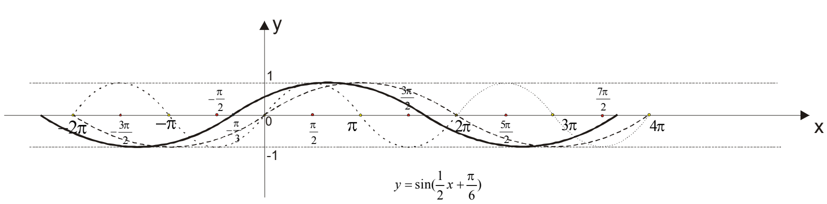
$$a = -2, b = \frac{1}{2}, c = \frac{\pi}{6} \rightarrow T = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi, \text{ so } \boxed{T = 4\pi} \text{ and } \frac{c}{b} = \frac{\frac{\pi}{6}}{\frac{1}{2}} = \frac{\pi}{3}, then\ \ is\ \ \boxed{\frac{c}{b} = \frac{\pi}{3}}$$



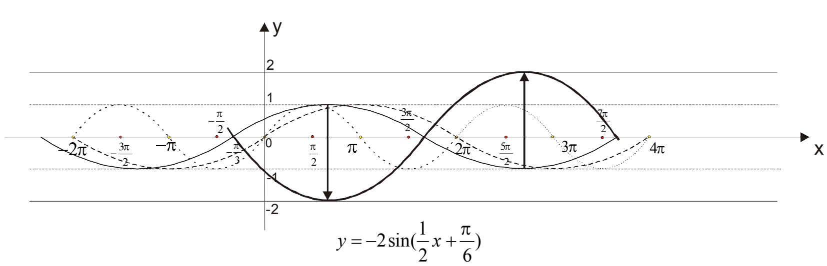
slika 1.



slika 2.



slika 3.



slika 4.

If we work through the examination:

Zero function

$y=0$

$-2\sin(\frac{1}{2}x+\frac{\pi}{6})=0$

$\sin(\frac{1}{2}x+\frac{\pi}{6})=0 \rightarrow \frac{1}{2}x+\frac{\pi}{6}=0 \vee \frac{1}{2}x+\frac{\pi}{6}=\pi$

$\frac{1}{2}x+\frac{\pi}{6}=0 \rightarrow \boxed{x=-\frac{\pi}{3}}$ and when we add the period: $\boxed{x=-\frac{\pi}{3}+4k\pi}$

$\frac{1}{2}x+\frac{\pi}{6}=\pi \rightarrow \boxed{x=\frac{5\pi}{3}}$ and when we add the period: $\boxed{x=\frac{5\pi}{3}+4k\pi}$

Maximum

$y=2$

$-2\sin(\frac{1}{2}x+\frac{\pi}{6})=2$

$\sin(\frac{1}{2}x+\frac{\pi}{6})=-1$

$\frac{1}{2}x+\frac{\pi}{6}=\frac{3\pi}{2}$

$\frac{1}{2}x=\frac{8\pi}{6}$

$x=\frac{8\pi}{3} \quad \rightarrow \quad \boxed{x=\frac{8\pi}{3}+4k\pi}$

Minimum

$y=-2$

$-2\sin(\frac{1}{2}x+\frac{\pi}{6})=-2$

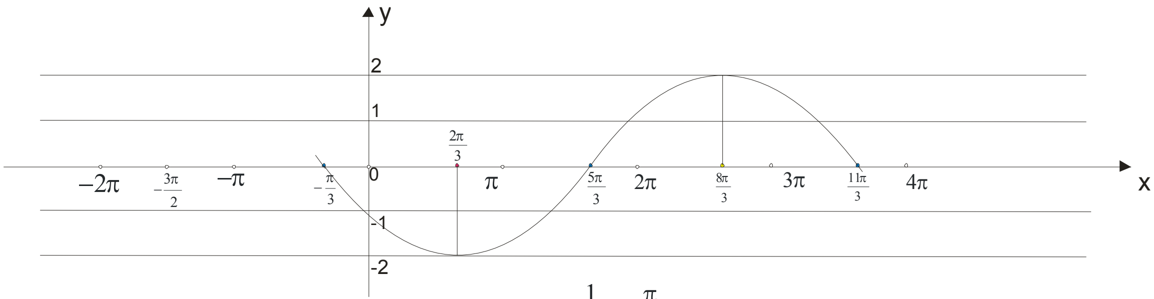
$\sin(\frac{1}{2}x+\frac{\pi}{6})=1$

$\frac{1}{2}x+\frac{\pi}{6}=\frac{\pi}{2}$

$\frac{1}{2}x=\frac{2\pi}{6}$

$x=\frac{2\pi}{3} \rightarrow \boxed{x=\frac{2\pi}{3}+4k\pi}$

To make a graph:



Example 3. $y = 2\cos(2x + \frac{\pi}{4})$

Solution:

Graph of this function is constructed in the same way as the sine function. The difference is only in the fact that the **initial** graph is $y = \cos x$

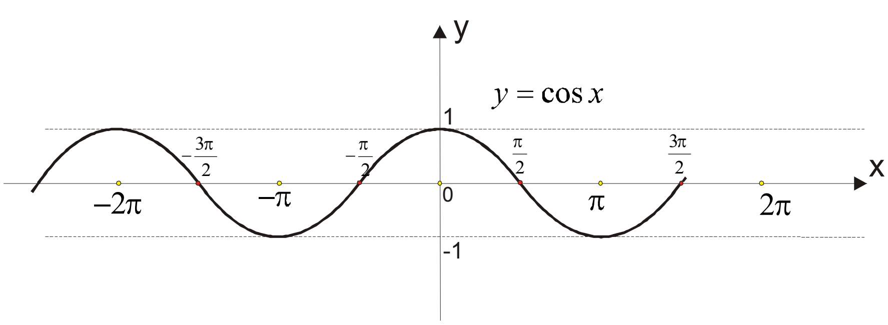
For $y = 2\cos(2x + \frac{\pi}{4})$ is:

$$a = 2, b = 2, c = \frac{\pi}{4}$$

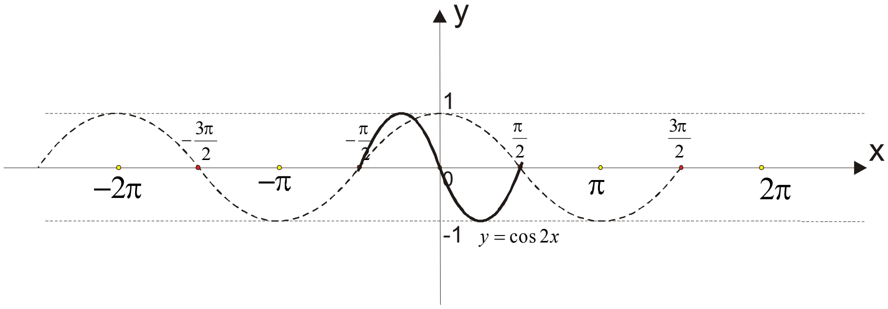
$$T = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi \to \boxed{T = \pi}$$

$$\frac{c}{b} = \frac{\frac{\pi}{4}}{2} = \frac{\pi}{8} \to \boxed{\frac{c}{b} = \frac{\pi}{8}}$$

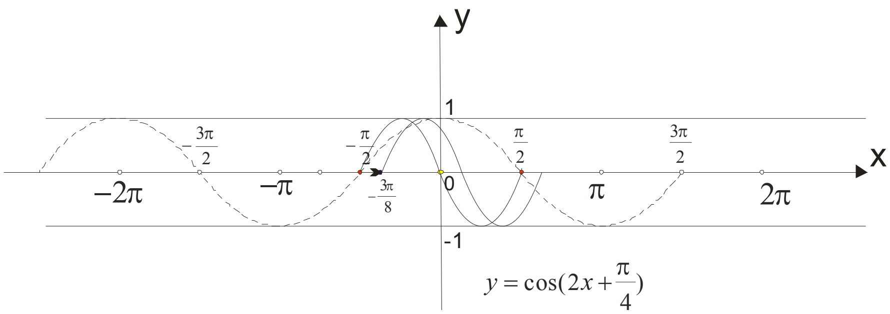
Move from graphic $y = \cos x$:



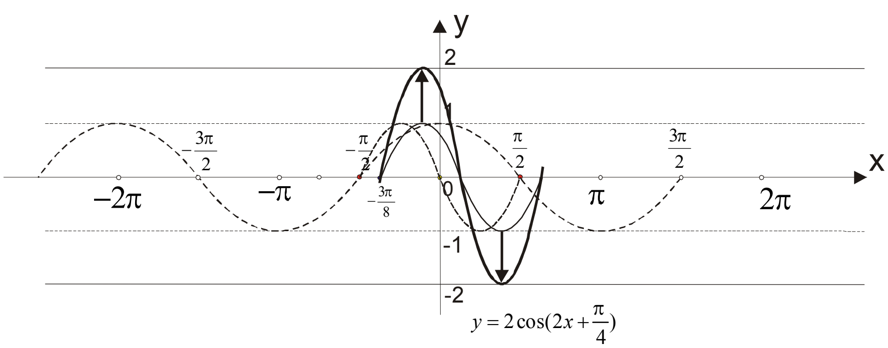
Further , we draw graph $y = \cos 2x$



$\frac{c}{b} = \frac{\pi}{8}$, We move the graphics for $\frac{\pi}{8}$ right :



Amplitude is *a=2.*



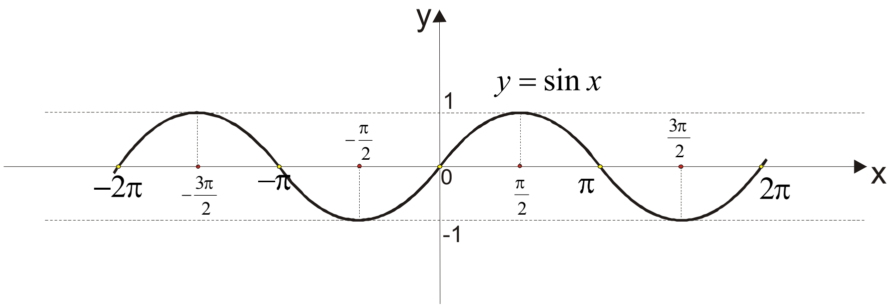
Here is the final graphic.

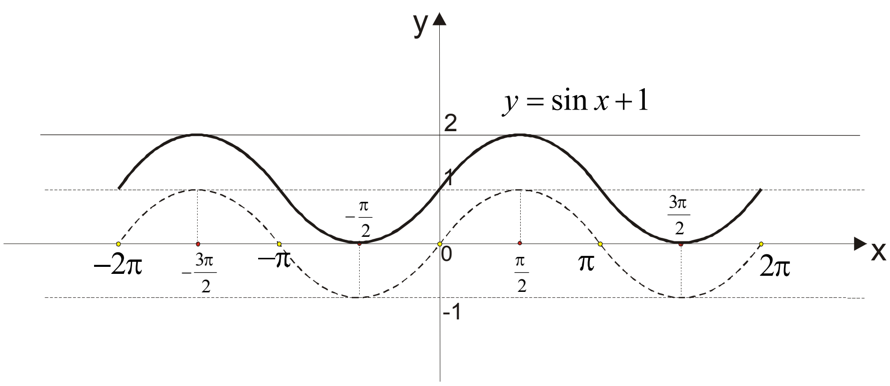
Example 4. $y = \sin x + 1$

Solution:

This situation until now we did not have ... But we did something similar with the quadratic function (see the file)

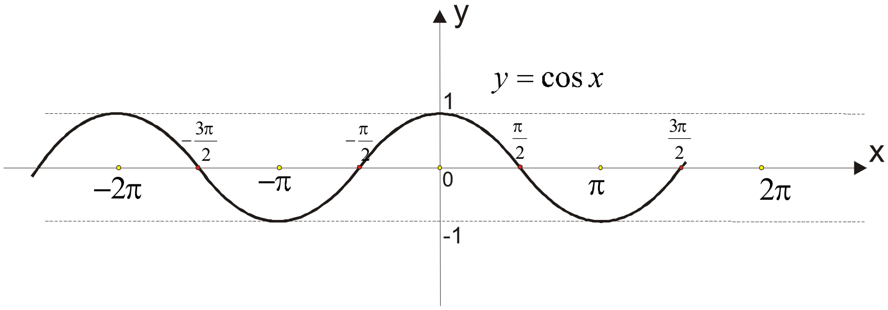
W e will draw a graph $y = \sin x$ and the whole plot to raise up to 1.

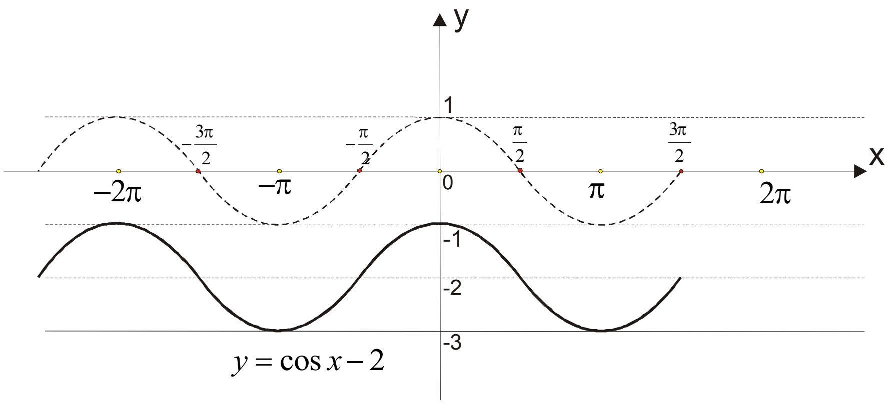




Example 5. $y = \cos x - 2$

Solution:





Example 6. **Draw a graph** $y = \sin x - \sqrt{3}\cos x$

<u>Solution:</u>

Here is our first job to "pack" function of the form $y = a\sin(bx + c)$ or $y = a\cos(bx + c)$.

$y = \sin x - \sqrt{3}\cos x$ ....add $\frac{2}{2}$

$y = \frac{2}{2}\sin x - \frac{2}{2}\sqrt{3}\cos x$

$y = 2(\frac{1}{2}\sin x - \frac{\sqrt{3}}{2}\cos x) \rightarrow$ we know that $\cos\frac{\pi}{3} = \frac{1}{2}$ and $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$, replace ...

$y = 2(\ \cos\frac{\pi}{3}\cdot\sin x - \sin\frac{\pi}{3}\cdot\cos x)$

$y = 2(\ \sin x\cdot\cos\frac{\pi}{3} - \cos x\cdot\sin\frac{\pi}{3})$ $\rightarrow$ *this is* formula $\sin(x - y) = \sin x\cos y - \cos x\sin y$

$$\boxed{y = 2\sin(x - \frac{\pi}{3})}$$

So, given the function $y = \sin x - \sqrt{3}\cos x$ we reduce to the shape $y = 2\sin(x - \frac{\pi}{3})$

We leave you to train yourself to try to construct…

Example 7. $y=\sin(2x-\frac{\pi}{4})+\cos(2x-\frac{3\pi}{4})$

Solution:

And here we have a tricky situation. We will use formula

$$\cos x=\sin(\frac{\pi}{2}-x)$$

$$y=\sin(2x-\frac{\pi}{4})+\cos(2x-\frac{3\pi}{4})$$
$$y=\sin(2x-\frac{\pi}{4})+\sin[\frac{\pi}{2}-(2x-\frac{3\pi}{4})]$$
$$y=\sin(2x-\frac{\pi}{4})+\sin[\frac{\pi}{2}-2x+\frac{3\pi}{4}]$$
$$y=\sin(2x-\frac{\pi}{4})+\sin(\frac{5\pi}{4}-2x)\rightarrow \text{ then we use : } \sin x+\sin y=2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$$
$$y=2\sin\frac{2x-\frac{\pi}{4}+\frac{5\pi}{4}-2x}{2}\cdot\cos\frac{2x-\frac{\pi}{4}-(\frac{5\pi}{4}-2x)}{2}$$
$$y=2\sin\frac{\cancel{2x}-\frac{\pi}{4}+\frac{5\pi}{4}\cancel{-2x}}{2}\cdot\cos\frac{2x-\frac{\pi}{4}-\frac{5\pi}{4}+2x}{2}$$
$$y=2\sin\frac{\pi}{2}\cdot\cos\frac{4x-\frac{3\pi}{2}}{2}\rightarrow \textit{we know that } \sin\frac{\pi}{2}=1$$
$$y=2\cdot 1\cdot\cos(\frac{4x}{2}-\frac{\frac{3\pi}{2}}{2})$$
$$\boxed{y=2\cdot\cos(2x-\frac{3\pi}{4})}$$

And this is for training ... If you are not getting along, send us an email and we will try to help, somehow…